

Introduction

Precalculus combines the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus, and strengthens students' conceptual understanding of problems and mathematical reasoning in solving problems. Facility with these topics is especially important for students intending to study calculus, physics, and other sciences, and/or engineering in college. Because the standards for this course are (+) standards, students selecting this Precalculus course should have already met the college and career ready standards.

For the high school Precalculus course, instructional time should focus on four critical areas: (1) extend work with complex numbers; (2) expand understanding of logarithms and exponential functions; (3) use characteristics of polynomial and rational functions to sketch graphs of those functions; and (4) perform operations with vectors.

- (1) Students continue their work with complex numbers. They perform arithmetic operations with complex numbers and represent them and the operations on the complex plane. Students investigate and identify the characteristics of the graphs of polar equations, using graphing tools. This includes classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.
- (2) Students expand their understanding of functions to include logarithmic and trigonometric functions. They investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. This includes the role of e , natural and common logarithms, laws of exponents and logarithms, and the solutions of logarithmic and exponential equations. Students model periodic phenomena with trigonometric functions and prove trigonometric identities. Other trigonometric topics include reviewing unit circle trigonometry, proving trigonometric identities, solving trigonometric equations, and graphing trigonometric functions.
- (3) Students investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. They determine zeros, upper and lower bounds, y -intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Students translate between the geometric description and equation of conic sections. They deepen their understanding of the Fundamental Theorem of Algebra.
- (4) Students perform operations with vectors in the coordinate plane and solve practical problems using vectors. This includes the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Overview

Number and Quantity

The Complex Number System

- Perform arithmetic operations with complex numbers.
- Represent complex numbers and their operations on the complex plane.

Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Algebra

Reasoning with Equations and Inequalities

- Solve systems of equations.

Functions

Interpreting Functions

- Analyze functions using different representations.

Building Functions

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Geometry

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section.

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for an express regularity in repeated reasoning.

Statistics and Probability

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems.
- Use probability to evaluate outcomes of decisions.

Content Standards

Number and Quantity

The Complex Number System

N-CN
Perform arithmetic operations with complex numbers.

- (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

- (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120° .*
- (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

- (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*
- (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities

N-VM
Represent and model with vector quantities.

- (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \mathbf{v} , $|\mathbf{v}|$, $\|\mathbf{v}\|$, v).
- (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

- (+) Add and subtract vectors.
 - (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
 - (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
 - (+) Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of \mathbf{w} , with the same magnitude as \mathbf{w} and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- (+) Multiply a vector by a scalar.
 - (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
 - (+) Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Arithmetic with Polynomials and Rational Expressions

A-APR

Use polynomial identities to solve problems.

5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Reasoning with Equations and Inequalities

A-REI

Solve systems of equations.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Functions

Interpreting Functions

F-IF

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Building Functions**F-BF****Build a function that models a relationship between two quantities.**

1. Write a function that describes a relationship between two quantities.
 - c. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.*

Build new functions from existing functions.

4. Find inverse functions.
 - b. (+) Verify by composition that one function is the inverse of another.
 - c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
 - d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Trigonometric Functions**F-TF****Extend the domain of trigonometric functions using the unit circle.**

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★

Prove and apply trigonometric identities.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Geometry**Similarity, Right Triangles, and Trigonometry****G-SRT****Apply trigonometry to general triangles.**

9. (+) Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles**G-C****Understand and apply theorems about circles.**

4. (+) Construct a tangent line from a point outside a given circle to the circle.

Expressing Geometric Properties with Equations**G-GPE****Translate between the geometric description and the equation for a conic section.**

3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Geometric Measurement and Dimension

G-GMD

Explain volume formulas and use them to solve problems.

2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Statistics and Probability

Using Probability to Make Decisions

S-MD

Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate the outcomes of decisions

5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
 - a. (+) Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.*
 - b. (+) Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident*